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Combined damage detection techniques

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Abstract

Linear damage detection techniques are used frequently because of their simplicity and their easy interpretation. In this paper, it will be shown however that linear techniques are not very robust with respect to environmental changes and interstructure variability. With the aid of experimental results it will be demonstrated that non-linear damage detection techniques, although being more complex, are less sensitive to these effects. In addition, two damage detection approaches will be proposed that combine the advantages of different classes of techniques. Firstly, a combined linear–non-linear approach is described. In the second proposed method, static and dynamic measurement techniques will be combined. Using experimental damage detection results, it will be shown that both proposed combined techniques are less sensitive to environmental changes while leading to easy interpretation of results. (© 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

The research in this paper has been developed in the framework of the SLAT TRACK project [1]. This project aims at predicting the life and redesigning critical airplane components like the slat track (which extends the surface of the wing during take-off and landing).

The validation of existing damage detection techniques in this paper, will be performed in view of the monitoring of slat tracks during fatigue loading. Because fatigue tests of a slat track take several weeks or even months, the environmental conditions will probably change. Moreover, the long testing time also implies that it should be possible to detect the damage in an automatic manner. Therefore, all applicable techniques should be as simple as possible (i.e., not involving a lot of tuning).

Damage detection using vibration measurements started several decades ago, and since then hundreds of publications have been written on the subject (see the overview in Ref. [2]). Some of

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these traditional techniques (not necessarily the most efficient, but rather the most widely accepted) are used as reference techniques and are discussed in brief in Section 2. The two proposed combined damage detection procedures are described in Section 3. Both the proposed and existing damage detection techniques are validated on several beams with fatigue cracks. The experimental set up which is used is given in Section 4, while the damage detection results can be found in Section 5.

2. Review of damage detection techniques

The existing damage detection techniques can be categorized in Sections 2.1 and 2.2.

2.1. Linear techniques

Linear damage detection techniques are based on the fact that changes in the modal parameters (resonance frequencies, damping values and mode shapes) can be related to damage. A major advantage of the techniques compared to other non-destructive testing techniques is that the approximate location of the fault should not be known in advance (they are global in contrast to more local techniques like ultrasonic inspection). Furthermore, measurements can be made in a reasonable time, with not too much user interaction.

A large number of different linear damage detection techniques—with varying sensitivity—is available in the literature (see the vast literature overview in Ref. [2]). In this paper the following standard techniques will be used for comparison:

- (1) changes of resonance frequencies [3];
- (2) changes of damping values;
- (3) modal assurance criterion (MAC) and co-ordinate-MAC (COMAC) values [4] (the MAC is nothing more than a correlation coefficient between 2 modes Ψ_1 and Ψ_2 : MAC(Ψ_1, Ψ_2) = $|\Psi_1^{\rm H}\Psi_2|^2/(\Psi_1^{\rm H}\Psi_1)\Psi_2^{\rm H}\Psi_2$, while the COMAC is the correlation of the vectors Φ^i and Φ^j formed by taking the values of all the modes at two degrees of freedom (d.o.f.) *i* and *j*: COMAC(Φ^i, Φ^j) = $|(\Phi^i)^{\rm H}\Phi^j|^2/((\Phi^i)^{\rm H}\Phi^i)(\Phi^j)^{\rm H}\Phi^j)$;
- (4) the strain energy method [5] which computes for each d.o.f. k the damage index $\beta_k = \sum_{i=1}^{N_m} f_{ik}^* / \sum_{i=1}^{N_m} f_{ik}$, where f_{ik} is given by $f_{ik} = \int_{a_k}^{a_{k+1}} (\delta^2 \Phi_i / \delta x^2)^2 / \int_0^l (\delta^2 \Phi_i / \delta x^2)^2$ with $[a_k, a_{k+1}]$ the interval in which the d.o.f. k is and l the length of the beam.

To minimize the effect of stochastic uncertainties (measurement noise), a maximum likelihood modal parameter estimation algorithm [6] was used.

A large number of problems occur when using linear damage detection techniques:

(1) the modal parameters also change when the environment (e.g., the temperature) changes;

(2) the influence of the measurement set up on the modal parameters is very large (the boundary conditions, the stinger which is used, etc.);

(3) a small change of the structure (e.g., geometric tolerances, fuel consumption) can lead to larger differences than the presence of a crack;

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(4) modal parameters are not very sensitive to damage (only large cracks can be detected by changes in resonance frequencies);

(5) it is difficult to make a statement on the severity of the damage (without a finite element model of the structure);

(6) often the structure exhibits some non-linearities, making the modal analysis difficult.

Statements (1)–(4) will be illustrated with the aid of an experimental example further on in the paper (see Sections 4 and 5).

As an alternative to linear damage detection techniques, non-linear features can be used. A short overview is given in the next section.

2.2. Non-linear techniques

Non-linear damage detection techniques are based on the fact that different damage scenarios (delaminations, cracks) give rise to a non-linear vibration behavior, while the intact structure is mainly linear (non-linearities in the intact structure occur but are usually smaller than the ones created by the damage).

Many non-linear detection techniques have been proposed in the literature: e.g., the Hilbert transform, the restoring force surface method, higher order frequency response functions, non-linear auto-regressive models, etc. (see Refs. [7,8] for a complete overview).

However, many of the existing non-linear detection techniques cannot be used straightforwardly to detect the presence of a crack. The reason for this is that the displacement must be relatively large in order to open and close the crack during the excitation. On the other hand, a high amplitude will result in structural non-linearities (in particular when using a signal with a flat spectrum). Therefore, a careful design of the excitation signal is needed. A particularly interesting excitation signal was introduced for non-linear damage detection purposes in Ref. [9]. The authors used a combination of a high-amplitude low-frequency sine to open and close the crack with a (lower-amplitude) higher frequency sine to detect inter-harmonic distortions which are created by the opening and closing of the crack. The results in Section 5 show that this so-called non-linear wave modulation spectroscopy (NWMS) technique works well.

An important disadvantage of the NWMS technique (as well as of most other non-linear damage detection techniques) is that a lot of tuning is required, since the method does not work well for two arbitrary modulation frequencies (both the selection of the high and low frequencies is important).

The main problem when considering the efficiency of damage detection techniques is that the success is dependent on the actual structure and the damage scenario that is chosen. For some applications linear techniques give sufficient information and non-linear techniques do not contribute, while for other cases linear techniques are much too insensitive and non-linear techniques offer a solution. To circumvent this problem two combined approaches will be proposed in the next section.

The combined linear–non-linear method (Section 3.1) estimates both linear features (modal parameters) and non-linear features in one single measurement. In this way damage can be detected even if no non-linearities are caused by the damage or if the resonance frequency changes are too small.

The combined static-dynamic technique (Section 3.2) allows one to compare modal parameter changes of intact and cracked structures without having the reference data of the intact state of the structure.

3. Combined approaches

3.1. Combined linear-non-linear

The proposed combined linear-non-linear method is based on a multi-sine excitation signal x(t)

$$x(t) = \sum_{i=0}^{N} X_i \sin(2\pi i f_0 t + \varphi_i),$$
(1)

with N being the number of excited frequencies and f_0 the fundamental frequency. The fundamental frequency f_0 is chosen such that enough excitation lines are present between two modes in order to be able to separate them (for most medium size structures $f_0 = 1$ Hz will be enough).

In order to detect a crack at the same time that a large-amplitude load (e.g., a fatigue load) is applied to open and close the crack, the multi-sine signal is split up into a low frequency $x_1(t)$ and a higher frequency $x_2(t)$ parts as

$$x(t) = x_1(t) + x_2(t) = \sum_{i=0}^{N_1} X_i^1 \sin(2\pi i f_0 t + \varphi_i^1) + \sum_{i=N_1+1}^{N} X_i^2 \sin(2\pi i f_0 t + \varphi_i^2),$$
(2)

with N_1 being typically a few tens. In order to detect damage using both linear and non-linear features the following choices are made:

- the phases φ_i¹ and φ_i² are chose uniformly randomly in [0, π];
 the energy X_i¹ for i = 1, ..., N₁ of the component at frequencies f₀, 2f₀, ..., N₁f₀ is taken high enough (resulting in a displacement in the order of magnitude of a few tenths of a mm). This will allow the crack to open and close. A typical example of such a low-frequency load signal $x_1(t)$ is an in-service fatigue load which is applied on a test rig;
- the energy X_{4i+1} with $i = \lfloor (N_1 1)/4 \rfloor$, ..., $\lfloor (N 1)/4 \rfloor$ of the signal $x_2(t)$ is taken lower than for the low-frequency signal $x_1(t)$ (this is to make sure that the intact structure will not vibrate non-linearly). From the measured forces and responses (e.g., displacements) at frequency lines $(4i+1)f_0$ for $i = \lfloor (N_1-1)/4 \rfloor$, ..., $\lfloor (N-1)/4 \rfloor$, one can extract modal parameters;
- the frequency lines in between $((4i+2)f_0 \text{ for } i = [(N_1-2)/4], ..., |(N-2)/4| \text{ and } (4i+3)f_0$ for $i = \lfloor (N_1 - 3)/4 \rfloor, ..., \lfloor (N - 3)/4 \rfloor$ and $(4i + 4)f_0$ for $i = \lfloor (N_1 - 4)/4 \rfloor, ..., \lfloor (N - 4)/4 \rfloor$ are not excited. If in the measurements of the force or the response some energy is present on those lines, this means that the structure behaves non-linearly. Moreover, it is also possible to separate even order (giving a response at frequencies 4i + 2 and 4i + 4) and odd order non-linearities (indicated by energy at components 4i + 3) as was shown in Ref. [10].

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The multi-sine which is used is an adaptation of the so-called odd-odd multi-sine [10]. Although the odd-odd multi-sine has been used with success for detecting damage in a composite beam [11], the straightforward use of a flat amplitude spectrum odd-odd multi-sine excitation did not give good results for the cracked beams in the experiments in the current paper (see Section 5). The adaptation, which consisted of the introduction of the low- and high-frequency parts, leads to the improvements:

- the crack opens and closes due to the large low-frequency displacement and can therefore more easily be detected;
- it becomes possible to apply the technique on-line with fatigue tests (the low-frequency part is the in-service load while the high-frequency part serves to detect linear and non-linear damage detection features).

A disadvantage of both linear and non-linear techniques (an also of the combined linear-nonlinear technique) is that a reference signal of an undamaged structure is needed. For linear techniques this is particularly true since modal parameters of an intact and a damaged structure have to be compared. For non-linear techniques one could say that damage is present as soon a non-linearity is detected. However, in real life all structures exhibit some amount of non-linearity. Therefore, non-linear techniques also require a comparison between an intact and a damaged state. This can be an important drawback, since usually the intact reference structure is no longer available, or the environmental conditions have changed significantly over the time of operation of the structure.

In the next section a second combined damage detection technique will be developed that does not require any comparison between intact and damaged structures. As is the case with the combined linear–non-linear technique, advantage will be made of the opening and closing of the crack when the structure is loaded.

3.2. Combined static-dynamic

The proposed combined static-dynamic damage detection technique includes the steps:

- (1) apply a DC pre-load to the structure;
- (2) use a multi-sine (with all frequencies except for the DC containing energy) to excite the structure;
- (3) estimate the modal parameters from the force and the response (e.g., acceleration) signals;
- (4) repeat steps (1)–(3) with other DC pre-load values;
- (5) compare the modal parameters obtained from different pre-loads.

The DC pre-load is used to make sure that the crack opens and closes for different DC offset values. By comparing the modal parameters at different DC offset values it is possible to detect the resonance frequency shift due to the opening and closing of the crack (and therefore qualify if a crack is present since no resonance frequency shift will be present if the beam is intact).

When applying the technique it is assumed that the structure is clamped; the crack is located between the excitation and the clamping location; the DC offset is large enough to open and close the crack.

More details on the combined static–dynamic technique are given below

Algorithm 1. Combined static-dynamic damage detection

FOR i = 1 to N_{DC} (with N_{DC} the number of applied DC steps, $N_{DC} \ge 2$)

Compute the DC offset V_{DC} as $V_{DC} = -V + 2V(i-1)/N_{DC} - 1$, with V the maximal DC offset.

Apply a flat spectrum multi-sine excitation (Eq. (1)) with dynamic amplitudes $X_k = V_{AC}$ for k = 1, ..., N and DC offset $X_0 = V_{DC}$.

Measure the force f(t) and response signals a(t) and calculate the FRFs.

Estimate the modal parameters (resonance frequency ω_i^j damping value ξ_i^j and mode shapes ψ_i^j where *i* denotes the current iteration step and $j = 1, ..., N_m$ with N_m the number of modes in the frequency band of interest) from the calculated FRFs $H(\omega)$, $H(\omega) = A(\omega)/F(\omega)$, with $A(\omega)$ and $F(\omega)$ the Fourier transforms of the force f(t) and response a(t), respectively. (To compute the modal parameters e.g., the technique presented in Ref. [6] can be used.)

END;

Compute the ratio of the modal parameters with respect to the modal parameters computed at the first iteration step (i.e. the first DC pre-load level):

$$\hat{\omega}_{i}^{j} = \omega_{i}^{j} / \omega_{1}^{j}$$
 for $j = 1, ..., N_{m}$, $\hat{\xi}_{i}^{j} = \xi_{i}^{j} / \xi_{1}^{j}$ for $j = 1, ..., N_{m}$, (3,4)

$$\hat{\psi}_{i}^{j} = \frac{\psi_{i}^{j}}{\psi_{1}^{j}}$$
 for $i = 1, ..., N_{m}$. (5)

When the structure is intact, the ratios $\hat{\omega}_i^j$, $\hat{\xi}_i^j$ and $\hat{\psi}_i^j$ should be equal to one. By tracking deviations from zero in the ratios $\hat{\omega}_i^j$, $\hat{\xi}_i^j$ and $\hat{\psi}_i^j$ it is possible to detect the presence of a crack.

4. Test set up

The validation experiment is performed on a steel beam (with dimensions: $5 \text{ mm} \times 500 \text{ mm}$). Fatigue cracks are produced by clamping the beam and exciting it at its second resonance frequency (127 Hz), at a nodal point of the mode (see Fig. 1). To monitor the vibration level of the beam during fatigue loading, a single-point laser vibrometer was used. As soon as the vibration amplitude decreased below a certain threshold, the fatigue loading was interrupted (*Note that the decrease in vibration amplitude is due to a decrease of the resonance frequencies which is caused by the crack*).

The measurements set up, which is used to validate the damage detection techniques, is shown in Fig. 2. The excitation force is measured using a force cell, while the response at 47 equidistant nodes on the beam was obtained with the aid of a Polytec[®] scanning laser vibrometer.

In total about 10 beams with different crack configurations were used in the validation. Because of space limitations only the results on two of them (i.e., Beam 1 with a small crack and Beam 2

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Fig. 1. Experimental set up of the fatigue loading to produce a crack.



Fig. 2. Experimental set up for the validation of damage detection techniques.



Fig. 3. Photographs of the four sides of the cracked Beam 1 using the liquid penetration technique.



Fig. 4. Photographs of the four sides of the cracked Beam 2 using the liquid penetration technique.

with a crack half way through) will be discussed in this paper. By visual inspection it was not possible to detect the cracks in any of the beams investigated (even if the location of the damage was known beforehand). Using the liquid penetration technique (with a Magnaflux[®] ZL37 sensitivity class 4 penetrant) the cracks could be visualized (see Figs. 3 and 4). The location of the crack was near node 8 for Beam 1 and between nodes 6 and 7 for Beam 2.

5. Results

For illustrative purposes the modal parameters of an intact beam (Beam 1) are shown in Table 1 and Fig. 5. From Fig. 5 it can be seen that the modes shapes are pretty smooth, which indicates that the measurement noise level is quite low.

The changes in modal parameters (and derivations thereof) due to damage are shown in Figs. 6 and 7 for Beams 1 and 2, respectively. From these figures it is clear that the resonance frequencies are not very sensitive to damage (a few percent deviation) while the damping values change several hundreds of percent. With respect to mode shape information, the MAC and COMAC values appeared to be useless as damage indicators (MAC values larger than 99% were obtained). This agrees with results which were obtained by other researchers [12].

The strain energy method gave a strain damage index which clearly indicated both the presence and the location of the crack (Figs. 6(e) and 7(e)). However, some tuning is required. In order to give acceptable results, the last two modes (which were quite noisy) could not included in the computation of the strain energy damage index (in Fig. 5 it can be seen that the estimated mode shapes are already quite smooth). Even when one ignores the most noisy mode shapes some spurious smaller peaks are present in the strain energy damage index (see Figs. 6(e) and 7(e)).

Table	1					
Mode	table	of	the	undamaged	steel	beam

Mode number	ω_r (Hz)	ξ_r (%)
1	21	0.3
2	127	0.05
3	375	0.3
4	733	1.2
5	1174	0.1
6	1825	0.3
7	2355	0.5



Fig. 5. First seven mode shapes of the steel beam.

Although some of the linear damage indicators were reasonably sensitive with respect to the fatigue crack (in particular the strain energy method), it is shown below that a false damage indication also results when the environmental conditions change. In Fig. 8 the changes in modal parameter between two measurements of the intact Beam 2 are given (the beam was taken out of the set up and repositioned afterwards). Fig. 9, on the other hand, shows the modal parameter changes due to a slight increase in environmental temperature (about 9°C). The results in Figs. 8 and 9 clearly show that linear damage detection techniques are very sensitive to environmental changes (i.e., false damage indications occur). Furthermore, differences (of resonance frequencies and damping values) of the same order of magnitude as the intact–cracked difference also result if two intact beams (with the same dimensions) are compared (compare Figs. 10 and 6). This illustrates that in order to use linear techniques, the undamaged state of the same beam has to be known. This is an important disadvantage of linear health monitoring techniques.



Fig. 6. Modal parameter deviations between intact and cracked Beam 1: (a) resonance frequencies; (b) damping values; (c) MAC values; (d) COMAC values; (e) strain energy method.

By looking at inter-harmonic distortions near the high frequency (see Figs. 11(b) and 12(b)) or at harmonics (see Figs. 11(b) and 12(b)) the intact and damaged beams can be separated. Note that also the intact beam measurement is non-linear (due to inherent structural non-linearities). This does not pose a problem, since non-linearities before and after damage are compared (this implies that the method is quite robust even if higher amplitudes are used). The non-linear distortions are much less sensitive to systematical errors. Indeed, the non-linear modulations for two different environmental temperatures are comparable (Fig. 13). The same is true for the nonlinear distortions for two different intact beams shown in Fig. 14. The method is very sensitive, as can be seen in Fig. 11. The fact that the non-linearity of Beam 1 (which has a smaller crack than Beam 2) is larger than for Beam 2 can be explained by taking into account that the crack in Beam 2 is open in unloaded condition (this can be seen in Fig. 4).



Fig. 7. Modal parameter deviations between intact and cracked Beam 2: (a) Resonance frequencies; (b) damping values; (c) MAC values; (d) COMAC values; (e) strain energy method.

The combined linear-non-linear technique (which is introduced in Section 3.1) was applied with a sine as the low-frequency signal $x_1(t)$ ($X_1^1 = 600$) and the energy of the low-frequency part was taken 600 times lower: $X_5^2 = 1$, $X_9^2 = 1$, $X_{13}^2 = 1$, ..., $X_{4093}^2 = 1$ (all other frequency lines are not excited). From Fig. 15 (in particular from the "non-linear features" in the zoom in Fig. 15(b)) it can be seen that the non-linear distortions of the cracked beam are up to 30 dB higher than for the intact beam. This implies the sensitivity of the combined linear-non-linear crack detection technique is very large compared to the linear features (which typically change a few percentages with the introduction of the crack). Although the "non-linear features" are very efficient in detecting the crack, they do not allow any localization of the damage. However, from the measurements at the excited lines (see the "linear features" in Fig. 15(b)) one could still extract modal parameters which can be used to locate the damage (e.g., with the strain energy index). In



Fig. 8. Modal parameter deviations between intact Beam 2 and intact Beam 2 repositioned: (a) resonance frequencies; (b) damping values; (c) strain energy method.



Fig. 9. Modal parameter deviations between intact Beam 2 and intact Beam 2 with an environmental temperature of 9° C higher: (a) resonance frequencies; (b) damping values; (c) strain energy method.



Fig. 10. Modal parameter deviations between intact Beam 1 and intact Beam 2: (a) resonance frequencies; (b) damping values; (c) strain energy method.



Fig. 11. Harmonic distortions for intact and cracked state of Beam 1: (a) inter-harmonic distortion around the high-frequency component; (b) second harmonic of the high frequency component.

this way the proposed combined linear-non-linear technique enables one to combine the robustness with respect to environmental changes of non-linear techniques, with the easy interpretation and localization typical for linear techniques.

The combined static–dynamic method (second proposed method, see Section 3.2) was applied with a maximal DC offset of V = 6 V and an excitation amplitude of $V_{AC} = 1$ V. Thirteen different DC steps were used (i.e., $N_{DC} = 13$). The resulting resonance frequency ratios $\hat{\omega}_i^j$



Fig. 12. Harmonic distortions for the intact and cracked states of Beam 2: (a) inter harmonic distortion around the high-frequency component; (b) second harmonic of the high-frequency component.



Fig. 13. Harmonic distortions for intact Beam 2 and intact Beam 2 with an environmental temperature increase of 9° C: (a) inter-harmonic distortion around the high-frequency component; (b) second harmonic of the high-frequency component.

(Fig. 16) and damping value ratios $\hat{\xi}_i^j$ (Fig. 17) demonstrate that a separation between intact and damaged beams was possible.

The resonance frequency ratios $\hat{\omega}_i^j$ of the intact beam (Fig. 16(a)) were very small (except for the first resonance frequency) indicating that no crack is present. The ratios for the cracked Beam 2 (Fig. 16(b)) increased up to 7%. Moreover, when comparing Figs. 16(b) and 7, it is can be seen that the resonance frequency ratios $\hat{\omega}_{13}^j$ (i.e., values at +6 V offset) agrees with the difference in resonance frequency between damaged and intact state of the beam. Therefore, using the proposed technique exactly the same results as with the traditional technique could be found without having to make any comparison with an intact structure.

In addition, when comparing the strain energy of the mode shapes at different offset levels (e.g., -5 and 2 V) the location of the damage could be discovered (see Fig. 18).



Fig. 14. Harmonic distortions for two different intact beams: (a) inter-harmonic distortion around the high-frequency component; (b) second harmonic of the high-frequency component.



Fig. 15. (a) Linear responses and non-linear distortions using an odd-odd multi-sine excitation for intact and cracked Beam 1; (b) zoom of (a).



Fig. 16. Scaled resonance frequencies for Beam 2 ((a) intact, (b) cracked state) under an increasing offset of the excitation signal.



Fig. 17. Scaled damping values for Beam 2 ((a) intact, (b) cracked state) under an increasing offset of the excitation signal.



Fig. 18. Strain damage index between -5 V offset and +2 V offset mode shapes of cracked Beam 2.

6. Conclusions

In this paper two combined approaches to detect faults in a cracked beam were proposed. It was shown that, in contrast to classical linear techniques, the introduced methods are not influenced by systematic errors like environmental uncertainties and inter-structure variability. Moreover, using the static-dynamic technique it was possible to detect damage without any reference to an intact beam.

In general it was noticed that the modal parameters (and in particular the resonance frequencies) only changed a few percentages, while the non-linear distortions changed more than an order of magnitude.

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